Power Requirements of Gas-Liquid Agitated Systems

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Power data for dispersion of air in liquids by means of a six-bladed flat blade turbine are presented in the form of a logarithmic plot of actual power consumed against a function of speed, impeller diameter, gas flow rate, and impeller power characteristics. The data are those of Michel (7), Bimbenet (2), Sachs (12), and Oyama and Endoh (10).

Turbines of from 3- to 8-in. diameter were run in tanks of from 6.5- to 18-in. diameter, with the D/T never exceeding 0.47. The fluids tested covered a density range of 0.8 to 1.65 g./ml. and a viscosity range of 0.9 to 100 centipoises.

Data are also presented on dispersion of air in a 50% by volume batch of carbon tetrachloride in water and dispersion of air in a suspension of Alundum particles in water, and compared with data on water in a similar system.

The qualitative effect of a surface active agent is demonstrated in a comparison of data for a 0.1% by weight mixture of Pluronic L-62 in water with data for pure water with the same apparatus.

In the field of agitation much work has been done to show that the power required by an agitator varies systematically with density of a homogeneous liquid (*11*) or an immiscible liquid pair (*8*) that is being agitated. The introduction of a gas into a liquid in which a mechanical agitator is rotating, on the other hand, brings about a mixture for which one is unable to calculate an average density significantly related to the power consumption of the impeller. The theory of power requirement of an impeller rotating in a homogeneous fluid is imperfectly developed, consisting mainly of the relatively unrigorous theory of models; that of an impeller handling a gas-liquid mixture has never been treated.

Gas-liquid contacting in mixing vessels has been investigated with regard to chemical reactions and mass transfer coefficients by Oldshue (*9*), Bartholomew, Karow, Sfat and Wilhelm (*1*) and others with no special attention being given correlation of power data. Cooper, Fernstrom, and Miller (*4*) reported that the agitation power with a high rate of gas feed to the impeller could be as little as 25% of the impeller's no-gas power but declared that the dependence of impeller power on gas rate is erratic and unpredictable. Foust, Mack, and Rushton (*5*) presented power data for an arrow-head disperser in the form of a plot of K (horsepower in water with air divided by horsepower in water at a constant speed) vs. the superficial air velocity in feet per second based on volume flow rate of gas and cross-sectional area of the tank. Data taken in air-liquid contacting with a standard flat-blade turbine (six blades) in water and viscous oils were presented by Sachs (*12*). A more recent investigation of gas-liquid agitation by Bimbenet (*2*) produced data on air-liquid contacting by use of the same kind of turbine in water and corn syrup solutions.

In 1955 Oyama and Endoh (*10*) attempted to correlate power in gas-liquid agitated systems by the K factor of Foust, Mack, and Rushton (*5*) and the dimensionless group $N_a = Q/ND^3$. They succeeded in a limited correlation, that of the effect of gas rate on the power of a particular impeller operating at constant speed. Later Calderbank (*3*) offered the same correlation to data obtained with a flat-blade turbine dispersing air in a number of liquids (density ranges 0.74 to 1.6 g./ml., viscosity range 0.5 to 28 centipoises). His plot of P_g/P_o against N_a at constant impeller speed resulted in two intersecting straight line sectors that only approximated the shape of the curve of Oyama and Endoh.

Previous to this, in 1955, Kalinske (*6*) discussed the power required by a flat-blade turbine operating in aerated sewage. He presented the data in the form of a plot of a term directly proportional to K vs. $(1 - N_a)$ and obtained a good correlation. Good agreement was reported on scale-up of the data from the 30-in. tank in which the experimentation was performed to a full scale sewage installation, but no quantitative comparison was made.

The investigation of power response that is reported here was undertaken in a search for understanding of the gas-dispersion phenomenon and increased reliability in the prediction of the power drawn by a gas-dispersing impeller. Mixco standard flat-blade turbines 3 and 4 in. in diameter were operated in flat-bottom cylindrical tanks, 6.5 and 11.4 in. in inside diameter, as indicated in Table 1. Both tanks were fully baffled. Metered air was introduced beneath the turbine by a sparger ring in the larger tank and by an open-end tube in the smaller. The authors' experience, supported by Oyama and Endoh, is that no difference in agitation power results from these different methods of gas supply. The impellers were driven by a variable-speed motor resting on a cradle type of dynamometer of established accuracy.

For additional details of apparatus and procedure and for the record of all the data taken the dissertation of Michel (*7*) should be consulted.*

DISCUSSION OF RESULTS

The data were first plotted as K against N_a. Unlike the data of Oyama and Endoh (*10*) and of Calderbank (*3*) they did not correlate as a straight line or as a smooth curve on either arithmetic or logarithmic coordinates. Instead they produced a family of curves, one for each gas rate (Figure 1).

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* Tabular material has been deposited as document 7077 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $3.75 for photoprints or $2.00 for 35-mm. microfilm.

Table 1. Summary of Systems Studied

Liquid	Density, (g./ml.)	Viscosity at operat. temp., (centipoise)	Tank diam., (in.)	Impeller diam, (in.)
Water	1.00	0.8	12	4
CCl_4	1.60	1.0	12	4
Glycerol [weak]	1.12	3.6	12	4
Glycerol [strong]	1.19	13.5	12	4
Mineral oil	0.87	28.0	12	4
CCl_4/water 50% by volume	1.30	—	12	4
Alundum/water	—	—	12	4
Water	1.00	0.8	12	3
Water	1.00	0.8	6.5	3
0.1 wt. % Pluronic L-62 in water	1.00	—	12	3

Range of variables
6.5-in. diameter tank
Gassed power—0.006 to 0.018 hp.
Gas rate—0.02 to 0.47 cu. ft./min.*
Liquid volume—0.92 gal.
Static liquid depth—6 in.
Turbine position—4 in. off bottom†

11.4-in. diameter tank
Gassed power—0.002 to 0.10 hp.
Gas rate—maximum 1.0 cu. ft./min.*
Liquid volume—4.4 gal.
Static liquid depth—9 in.
Turbine position—3 in. off bottom

* At impeller condition.
† Clearance measured from tank bottom to center plane of turbine, turbine coaxial with tank.

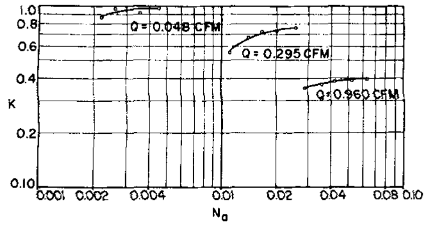

Fig. 1. K vs. N_a for the 4-in. turbine operating in the 12-in. tank filled with water.

The failure of the data to yield to the simple correlation of Oyama and Endoh may be the result of the relatively broad extent of experimental conditions investigated. The range of power values measured in this study was fifteenfold, a consequence of wide variation in both gas rate and impeller speed. Oyama and Endoh on the other hand held their impeller speed constant and varied their gas rate such that a twofold span of power values resulted. Thus their curve is analogous to any that might be drawn through points of constant speed in Figure 1.

In the absence of fruitful theory a search was begun for an empirical correlation that would successfully relate impeller power to speed, gas rate, fluid properties, and scale size over an extensive range of these variables. Logarithmic crossplots of N_a against P_g at constant K and of N_a against K at constant P_g were linear, indicating that

$$N_a \propto P_g^{-0.33} K^{-2}$$

or, by rearrangement

$$P_g \propto [P_o^2/N_a]^{0.43}$$

Figure 2 shows a logarithmic plot of P_g against P_o^2/N_a for a typical set of data, those for the system carbon tetrachloride/air being agitated by a 4-in. turbine in a 12-in. tank at three different air rates. The data for each air rate form a separate line, the lines being substantially parallel with slopes between 0.40 and 0.42.

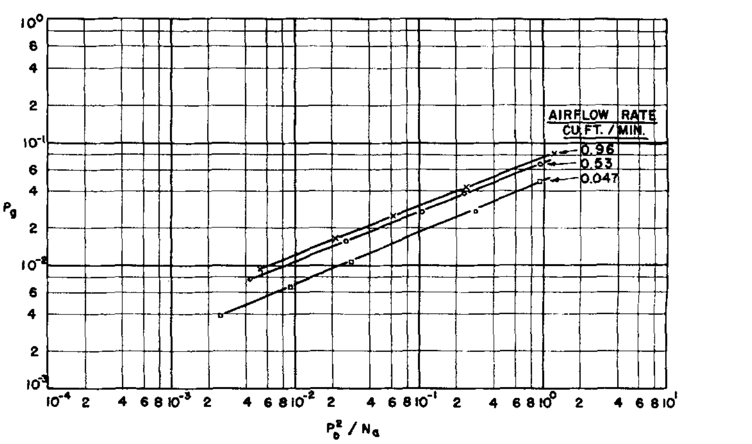

Fig. 2. P_g vs. P_o^2/N_a for the 4-in. turbine operating in the 12-in. tank filled with carbon tetrachloride.

A cross plot of P_g against Q (Figure 3) shows that $P_g \propto Q^{0.19}$. This relationship in combination with the preceding one indicates

$$P_g \propto (P_o^2 Q^{0.44}/N_a)^{0.43}$$

or

$$P_g \propto (P_o^2 N D^3/Q^{0.56})^{0.43}$$

A logarithmic plot of P_g against $(P_o^2 N D^3/Q^{0.56})$ for the data obtained in the 12-in. tank with the 4-in. turbine is shown in Figure 4. Included are data for air dispersed in water, carbon tetrachloride, mineral oil, and two strengths of glycerine solution (Table 1), covering three orders of magnitude of the abscissa quantity.* A line of slope 0.43 represents all these data within ±17%. The relatively fewer data collected for the system water/air with a 3-in. turbine in 12- and 6.5-in. vessels also correlate as straight lines of the same slope but with different intercepts, Figure 5. The line for the 3-in. impeller in the 12-in. tank lies almost within the spread of the data for the 4-in. turbine (about 20% above the 4-in.); the line for the 6.5-in. tank is about 35% below the latter.

Also shown for comparison in Figure 5 are the data of other investigators: those of Oyama and Endoh

* Actually only about one-third of the more than 400 data points measured and calculated are shown on the graph, because many of them overlay one another. The extreme points are all plotted however.

(*10*) for water/air in a 10.8-in. tank with a 3.6-in. turbine, those of Bimbenet (*2*) for air dispersed in water and corn syrup (viscosity from 1 to 90 centipoises) by 3- and 4-in. turbines in a 12-in. tank and by 5- to 8-in. turbines in an 18-in. tank, and those of Sachs (*12*) for air dispersed in water and 105-centipoise oil by a 6-in. turbine in an 18-in. tank. Inasmuch as none of these three sources reported no-gas power data, values for P_o had to be estimated before the abscissa group could be computed. The method of estimation was that of Rushton, Costich, and Everett (*11*), who reported a power correlation directly applicable to the turbines used by Bimbenet and Sachs and suggested correction factors for differences in turbine proportions that permit estimation of the power requirements of the impeller used by Oyama and Endoh. Remarkable as it may seem, a single line of slope 0.45 will represent within tolerable engineering limits the behavior of air-liquid systems involving liquid densities between 0.9 and 1.6 g./ml. and liquid viscosities between 1 and 100 centipoises in a one and one-halffold range of tank size and a range of impeller size such that impeller-to-tank diameter ratios lie between 0.25 and 0.44. It is likely that a wider range of liquid viscosity could be correlated in large equipment, that is with the agitation in the turbulent regime. The equation of this line constitutes a relationship by which the power required by a gas-dispersing agitator can be calculated:

$$P_g = c\left(\frac{P_o^2\,Q^{0.44}}{N_a}\right)^{0.45} = c\left(\frac{P_o^2 N D^3}{Q^{0.56}}\right)^{0.45}$$

The value of the coefficient c probably depends importantly on the geometry of the impeller; for the flat-blade turbine used in this study the authors' data, Bimbenet's, and Oyama's indicate that c is 0.08 when the units are horsepower, revolutions per minute, feet, and cubic feet per minute. The exponents of Q and of the group $[P_o^2 Q^{0.44}/N_a]$ also may be functions of equipment geometry, but probably less sensitively so than c.

The closeness to 0.5 of the slopes of all the lines plotted tempts one to conclude that P_g/P_o should be a power function of $ND^3/Q^{0.56}$, a relationship much more attractive for scale-up calculations. An effort to correlate the data of Bimbenet and the authors' water/air data in this fashion was not successful.

The results of two experiments involving three-phase systems are included in Figure 4. Air was dispersed into a 50-50 volume % mixture of water and carbon tetrachloride and into a slurry of fine (120 mesh) Alundum particles in water, the 4-in. turbine and 12-in. tank being used. The exact composition of the Alundum slurry is unknown, but the solids content was on the order of 5 wt. % The data fall well within the band of the authors' two-phase data, with no discernible systematic segregation. The impeller speed range, hence the power range, used in these experiments was considerably less than in the two-phase experiments; power values lay within a fourfold spread. This narrower experimental span was imposed by the lowest speed that would permit a satisfactory liquid-liquid or liquid-solid suspension.

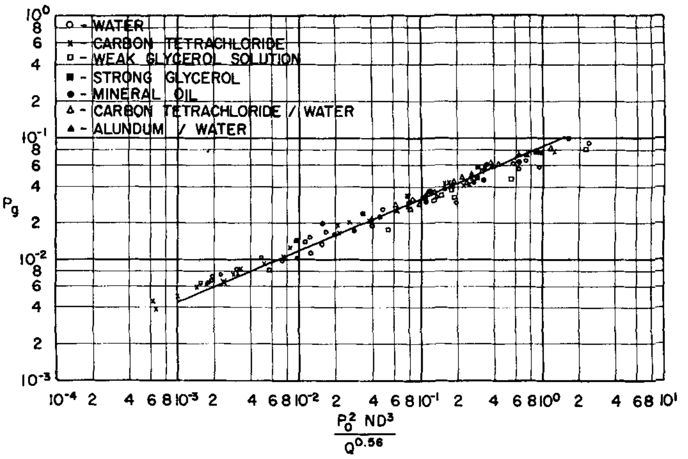

Fig. 4. P_g vs. $P_o^2ND^3/Q^{0.56}$ for the 4-in. turbine in the 12-in. tank.

The Effect of Interfacial Tension

The liquid-gas interfacial tension of the two-phase systems investigated in this research ranged from 27 to 72 dynes/cm. One might expect the effect of interfacial tension to be reflected in the Weber number $N_{We} = D^3N^2\rho/\sigma$. The extremes of the value for N_{We} encountered were 1.2×10^4 for water/air at lowest impeller speed and 3.6×10^5 for carbon tetrachloride/air at highest impeller speed, a thirtyfold range. The data for these two systems are correlated by the same line in Figure 4, indicating no effect of interfacial tension on impeller power.

Because of the industrial importance of liquid-gas suspensions containing a surface-active agent, in one experiment 0.1% of Pluronic L-62 brand surface-active agent was added to water, and a run was made with the 3-in. turbine in the 12-in. tank. As shown in Figure 6 the resulting data lie appreciably below the corresponding line for water/air, the value of P_g observed being as low as 60% of that expected. The surface tension of the Pluronic solution reported by the manufacturer of the agent is 37 dynes/cm., corresponding to a maximum value of $N_{We} = 2.4 \times 10^6$. Inasmuch as this is only two-thirds of the extreme value of N_{We} for carbon tetrachloride/air, the nonconformity of the data to the general correlation may reflect the uncertainty of surface-ten-

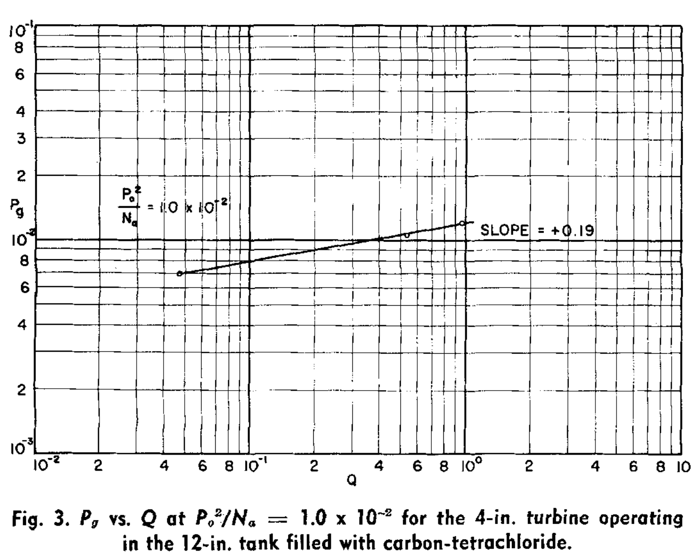

Fig. 3. P_g vs. Q at $P_o^2/N_a = 1.0 \times 10^{-2}$ for the 4-in. turbine operating in the 12-in. tank filled with carbon-tetrachloride.

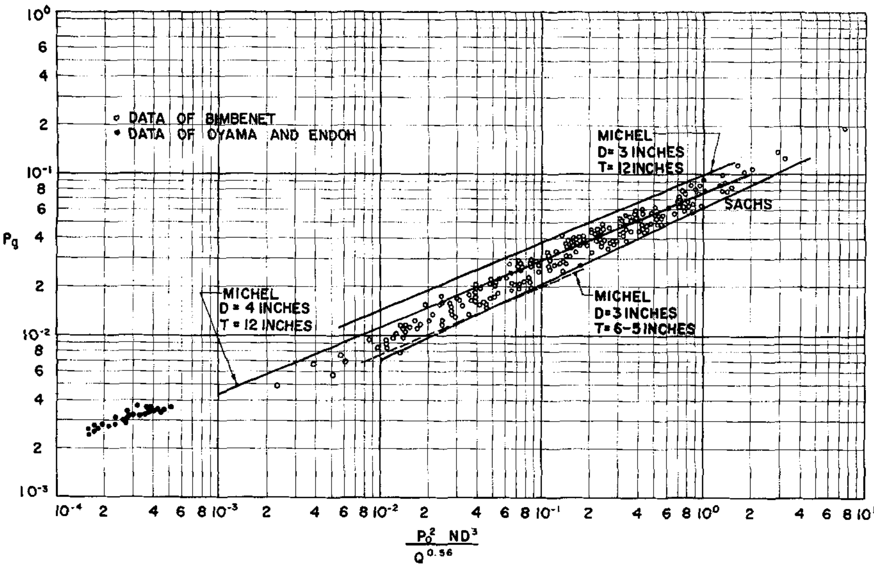

Fig. 5. ***P_g* vs. *$P_o^2ND^3/Q^{0.56}$* for data of Oyama and Endoh (*10*) and Bimbenet (2). Representations of the data of Michel (*7*) and Sachs (*12*).**

sion observations for solutions of surface-active materials. In any event the introduction of a surface-active agent may affect the power of a gas-dispersing agitator in a way not reliably predictable from the general correlation reported in this paper.

CONCLUSIONS

The power required by an agitator impeller dispersing gas into a liquid is usually less than the power required by the impeller operating at the same speed in a gas-free liquid. For equipment of a single size or of a narrow range of scale sizes, the power (P_g) can be calculated in terms of the no-gas power (P_o); thus

$$P_g = c\left(\frac{P_o^2 ND^3}{Q^{0.56}}\right)^{0.45}$$

This empirical equation involves absolutes rather than dimensionless ratios and does not necessarily hold for all sizes of equipment, even if scale models are considered. It obviously fails for extreme values of Q; as Q approaches zero, P_g should approach P_o, and as Q becomes very large, P_g should approach the value required by the impeller rotating in liquid-free air (a small but finite limit).

The equation therefore must be used cautiously as a scale-up aid and with extreme values of the independent variables employed. Furthermore, although it holds in form for aerated solutions of surfactants, the coefficient and exponent may have values different from those applicable to systems containing no surface-active agent.

Ungrounded in theory and unsuited for reliable extrapolation as the relationship is, it nevertheless provides the most general and successful relationship yet proposed between the power required by a gas-dispersing impeller and the factors on which that power depends. It embraces a much wider range of experimental conditions than the earlier correlations of Oyama and of Kalinske, and it avoids the practical unknown of gas holdup or suspension density required by Foust and co-workers. Insofar as the equation applies with a single set of coefficient and exponent values to a range of impeller sizes in a given tank, as it appears roughly to do, it defines quantitatively an interesting qualitative observation long familiar to those experienced with agitator dispersers; the smaller the impeller selected to deliver a particular amount of power in gas-free liquid, hence the higher the impeller speed, the greater the reduction in power when gas is supplied to the impeller.

To establish a firmer basis for design calculations a better understanding of the theory underlying the behavior of a gas-dispersing agitator and more extensive data on the performance of such agitators are required. Investigations in the turbulent range should be extended to involve plant-size tanks and the range of impeller-to-tank diameter ratios that may be encountered in practice (for example 0.2 to 0.7). The role of surfactants also needs delineation. The authors are continuing both theoretical and experimental study of the gas-dispersion phenomenon.

NOTATION

- c = constant
- D = impeller diameter, in. or ft.
- K = dimensionless, P_g/P_o at equal speeds
- N = rotational speed of impeller, rev./min.
- N_a = dimensionless ratio, Q/ND^3
- N_{We} = Weber number, $(D^3N^2\rho)/\sigma$, dimensionless
- P_g = power consumed in the gassed liquid, hp.
- P_o = power consumed in the ungassed liquid, hp.
- Q = volumetric flow rate of gas, cu.ft./min at temperature of liquid and static pressure at the impeller
- $\left(\frac{P_o^2 ND^3}{Q^{0.56}}\right)$ = correlating factor, $\left[\frac{\text{hp.}^2 \times \frac{1}{\text{min.}} \times \text{cu. ft.}}{[\text{cu. ft./min.}]^{0.56}}\right]$
- ρ = density
- σ = interfacial tension

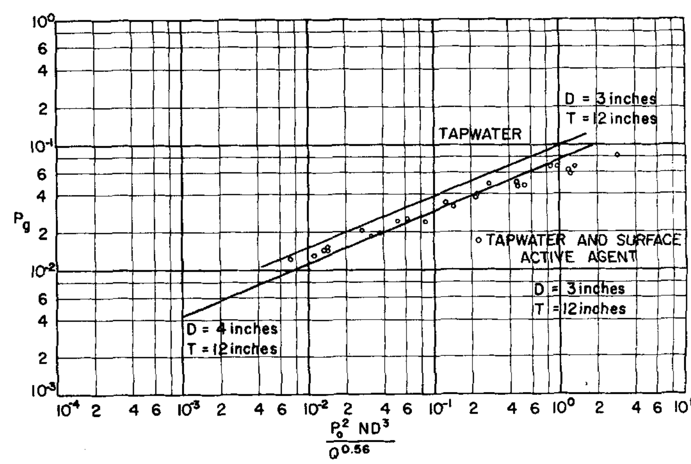

Fig. 6. ***P_g* vs. *$P_o^2ND^3/Q^{0.56}$* for data on 0.1% Pluronic L-62 in water compared with the general line for a 4-in. turbine in a 12-in. tank and with the line representing the 3-in. turbine operating in the 12-in. tank filled with water.**

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Transport Characteristics of Suspensions: Part IV. Friction Loss of Concentrated-Flocculated Suspensions in Turbulent Flow

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Turbulent flow friction factors were determined for flocculated suspensions of thoria, kaolin, and titania in tubes ⅛- to 1-in. diameter. The non-Newtonian laminar flow data were arbitrarily fitted with the Bingham plastic model. With this model the range of yield stress values was 0.018 to 1.39 lb.$_f$/sq. ft., with a maximum ratio of coefficient of rigidity to viscosity of suspending medium of 11.1. The volume fraction solids were varied from 0.042 to 0.23.

Two types of behavior were observed depending on the value of the yield stress. For yield values less than 0.5 lb.$_f$/sq. ft. the turbulent friction factors were always less than those for Newtonian fluids but tended to approach the Newtonian values as the Reynolds number was increased. For yield values greater than 0.5 lb.$_f$/sq. ft. the friction factors were again less than those for Newtonian fluids but tended to diverge from the Newtonian values as the Reynolds number was increased. Both sets of data were correlated with the Blasius relation with the coefficient and exponent given in terms of the laminar flow properties and the volume fraction solids.

The mechanics of the flow of solids suspended in liquids has attracted increased attention during the last decade, encompassing not only basic research but also applied studies directed toward large-scale operations, such as the transport of coal in commercial pipelines (*1*), and more unusual applications such as high energy solid additives to jet fuels (*2*) and the transport of fertile material in homogeneous nuclear reactors (*3*). An important aspect of the applied problems is the friction loss characteristics of the suspensions in turbulent flow. Although the primary measurements required for the determination of the friction loss characteristics (that is the pressure drop as a function of flow rate) are relatively simple to make, the subsequent interpretation and correlation are complicated by several factors. Not the least of these is that friction-loss correlations for Newtonian fluids are primarily empirical in nature. Although it has been established experimentally for Newtonian fluids that Reynolds number similarity gives similarity of friction factors, no equivalent solutions of the exact equations of motion are available. This means that in order to compare the data on the characteristics of suspension friction loss with data for ordinary fluids, great care must be exercised in the selection of the Reynolds number in order to insure similarity. The ambiguity in the determination of a suitable Reynolds number for suspensions arises because of the effect of the solid phase on the laminar viscosity. Particle size and shape, as well as solids concentration, are the principal factors affecting the viscosity. Suspensions of large symmetrically shaped particles (approximately 50 μ or larger) have Newtonian flow characteristics with viscosity a function of volume fraction solids (*4*). Suspensions of smaller particles or of assymetrically shaped particles (platelike or needlelike) possess non-Newtonian flow characteristics with apparent viscosities a function of the rate of shear, as well as particle size, shape and concentration (*5*). Thus possible viscosities which must be considered for use in suspension friction loss correlations include:

1. The viscosity of the suspending medium.
2. The suspension viscosity, a function only of solids concentration for Newtonian suspensions.
3. The apparent viscosity, a function of both shear rate and solids size, shape and concentration for non-Newtonian suspensions.
4. The limiting viscosity at high rates of shear, a function only of solids

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